OBVIS

Bridge of the Month No23 November 2012
Gothic arches

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Yes, I know that it is very nearly Christmas, but it has been a very busy year end with a trip to New Zealand in Novemember and a very urgent and all-consuming job from the day I got back. I don't have time to format this as an email so it will go as a link for download.

This month we will return to the Exe bridge as an example and think about the implications of pointed ribbed arches when it comes to assessment.

A bit of history

In Roman and Romanesque architecture arches were essentially semi-circular. This imposed a considerable limitation on the construction of vaults, though the romans overcame this by using square cross vaults. By 1100 at least, masons had grown bold with almost free form shapes of thin masonry. I don't think we have any way of reconstructing the thought processes involved.

A typical "gothic" vault has diagonal ribs that are close to semi-circular. To allow them to rise to the same level, the window heads and square ribs are pointed, though still essentially circular curves. The mason's learnt how to set these curves out using only geometrical constructions.

Groin vaults, those with no rib at the intersection of the webs, work well enough, as the Roman's knew well, but there is some tendency for the brick or stone on this angle to unravel through lack of bond and lack of compressive thrust. This is also the problem that demands complex coursing in skew arch barrels.

The masons found that they could build ribs first and then construct the webs without further formwork, using only a board cut to profile to introduce a slight transverse arch in the web. As each course was completed, it became self-supporting as an arch so there was only ever one stone held by the stiction of the mortar.

The savings in formwork of this conctruction were huge, because ribs could be erected very quickly and the formwork removed immediately. Thus only one set of rib forms was required for a cathedral.

The ribs usually had bosses at the intersections. Peter Dare (somtime Master Mason of Brisbane Cathedral) tells me that the centres are erected and the boss positioned first, then string lines erected from the boss to a point on the wall vertically above the centre line of the rib. The rib stones can then be positioned on the correct line and at the correct angle. If the mortar used is in very thin joints, or is itself stiff (for example made with coarse sand), the centre can be struck immediately the last stone is placed.

Ribbed arch bridges

The reasoning behind ribbed arch bridges is very similar to that of ribbed vaults. Ribs can be light and quickly built then the narrow centre can be moved across and a second rib constructed. A web, or barrel, can then be built on top of the ribs without further centring.

In the south of England, rivers are largely relatively slow moving and the engineers were able to build stocky piers often with the thickness greater than half the span. At the Mediaeval bridge in Exeter the piers and spans are essentially identical. On such stocky piers balancing forces is unnecessary and the arches can be built progressively through the bridge. In the hilly north, flows tend to be greater and faster so piers were made thinner and spans often much greater. Devils Bridge at Kirkby Lonsdale is one of the grandest. http://www.engineering-timelines.com/scripts/engineeringItem.asp?id=373 The larger arches span 16.7m or about 55ft. The piers are about 1/3 of that.

Figure 1 Devils Bridge Kirkby Lonsdale. Thanks to http://www.flickr.com/photos/chrispheath/

Ribbed and pointed arches

But to return to Exeter, this note is prompted by a question from an Archie user which raised issues over how these arches work.

The first issue is basic geometry. Old pointed bridges were frequently built with circular curves rising vertically from the face of the abutment or pier. If you are measuring a bridge like this for assessment, a deal of care is needed. The very gentle vertical rise from the springing means thatpicking the point of measurement can be very difficult. I would suggest taking the more obvious springing of the layer above the rib then fitting the rib below it. The geometry of this curve is not easy to find. In Archie we use the very basic span/rise/quarter point rise dimensions and fit a circular curve to the three points in arch half span.

For myself, I would prefer to begin with a bit more data and play with it in Excel. Make a decent xy plot of the points then superimpose a circular curve and juggle with the data till it fits (or doesn't, which then raises other issues.

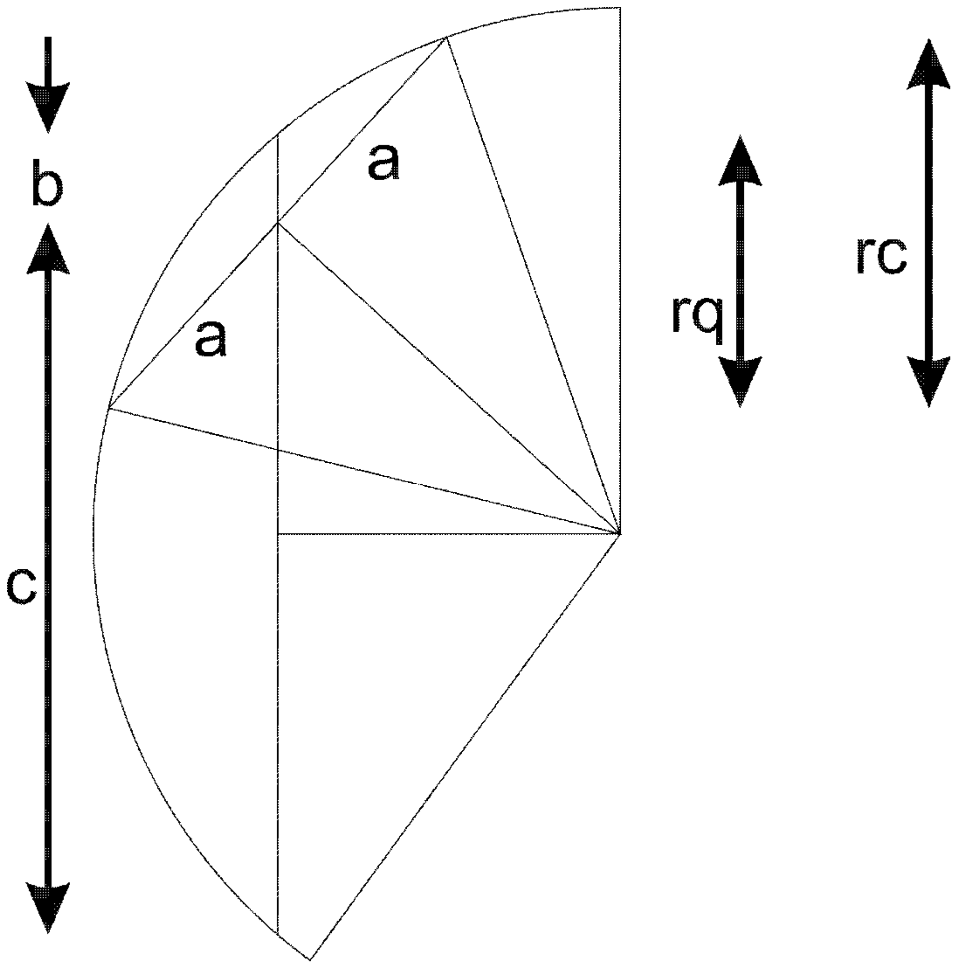
Figure 2 Exe bridge pointed span with circular curves

Of course, to set it out correctly we need a square photograph. I don't have one of this span but here is a nearby one.

The dimensions here are approximate but good enough for demonstration purposes.

Calculating the curves

Intersecting chords in a circle create similar triangles. The ratios of the sides can thus be used to compute the unknown length of a second chord and from that the radius and position of the centre of the circle. In a pointed arch the known chord is the diagonal from the springing to the crown. The unknown one is the vertical passing through the arch quarter point.



The simple rule here is that a*a=b*c. b = rq-0.5*rc so we can calculate C and thus b+c the vertical chord length. Half way down that cord is the level of the arch centre. The horizontal distance from the vertical chord to the centre is computed using the normal to the original chord and the drop from the centre of the chord to the centre line of the circle.

Analytical issues

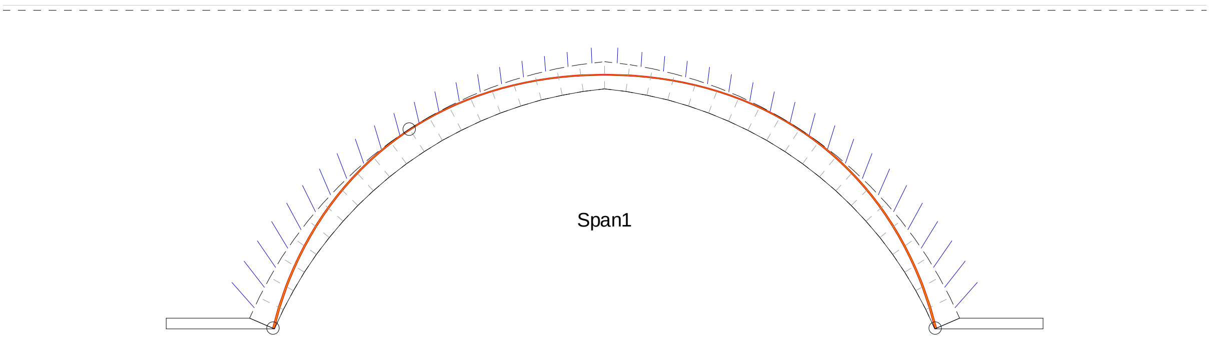
In real bridges, thrust lines always follow smooth curves. Pointed arches are therefore not obviously happy structures. In cathedrals, the issues are resolved by placing a heavy weight at the crown but this is obviously not possible in a bridge. The alternative is still fill. Pointed arches in arcades in cathedrals do not have bosses, but they are held firmly in shape by the wall above and the same is probably true in bridges.

Under the spandrel walls, the action is obvious but under the carriageway, construction is well hidden and behaviour more difficult.

My approach to questions like this is to ask what is possible before considering what confidence I have.

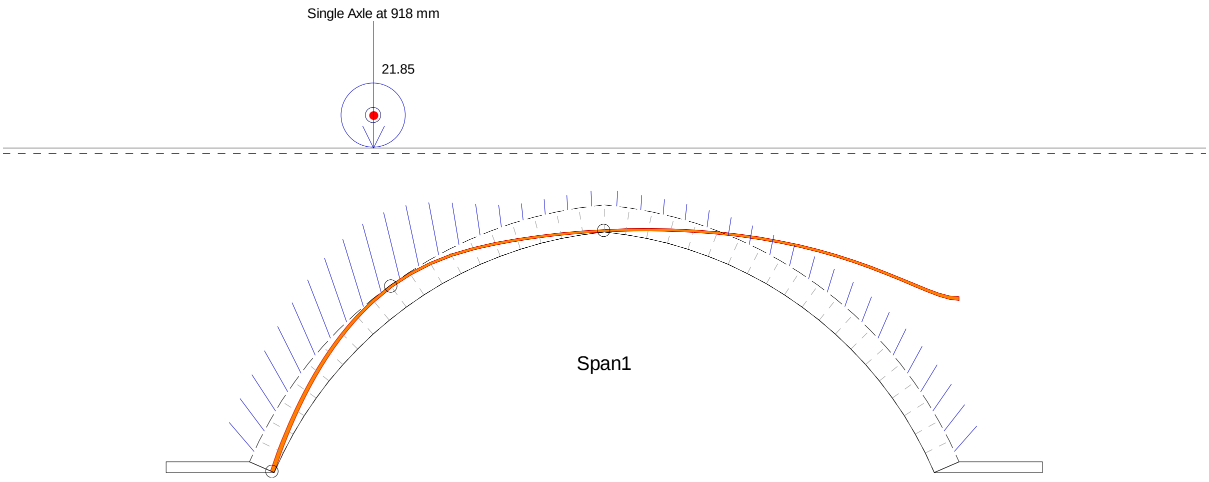
Continuing with the Exe bridge. The dimensions shown are directly measured off a photograph. We need a baseline for the span, which in this case is about 20ft or 6.1m

Using these basic dimensions produces an Archie model



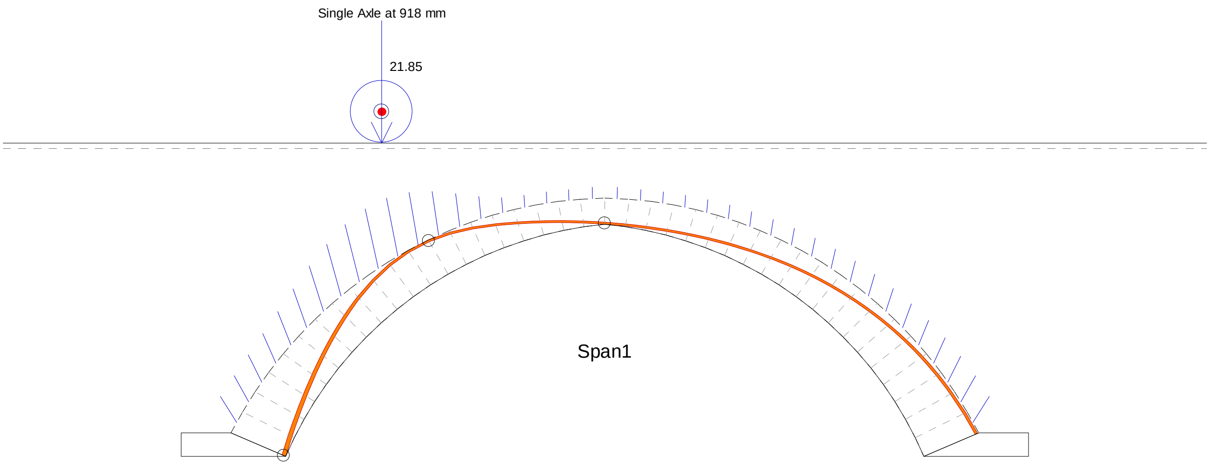
Note how the thrust passes flat over the crown of the arch and does not follow the pointed curve.

Putting a single modern axle on this model produces a quite unacceptable thrustline.



So that now it is quite clear that the arch cannot work if analysed simplistically using the models recommended in assessment codes. The question is, what might be wrong.

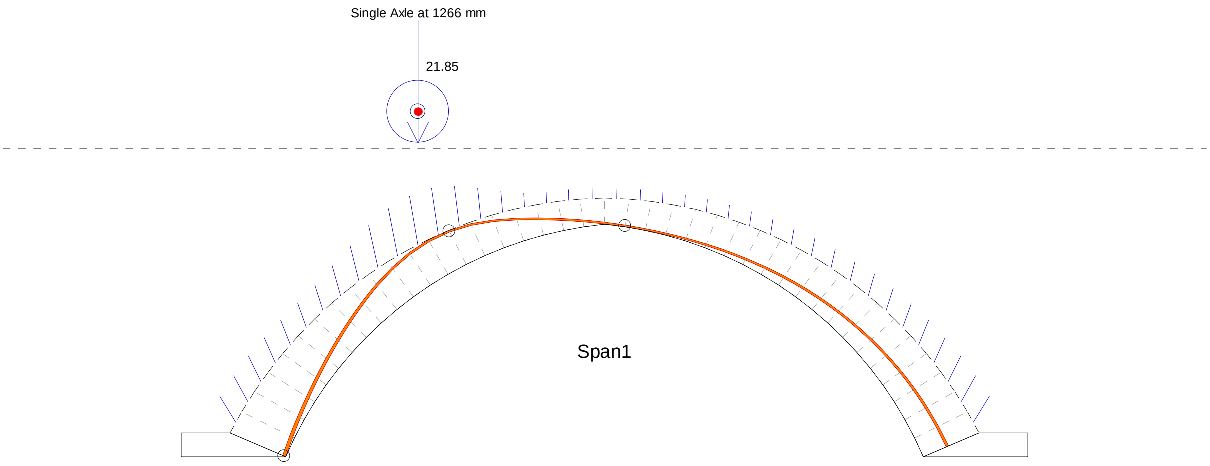
If the arch is built in such a way that the thickness increases towards the springings, the utcome is muchhappier.



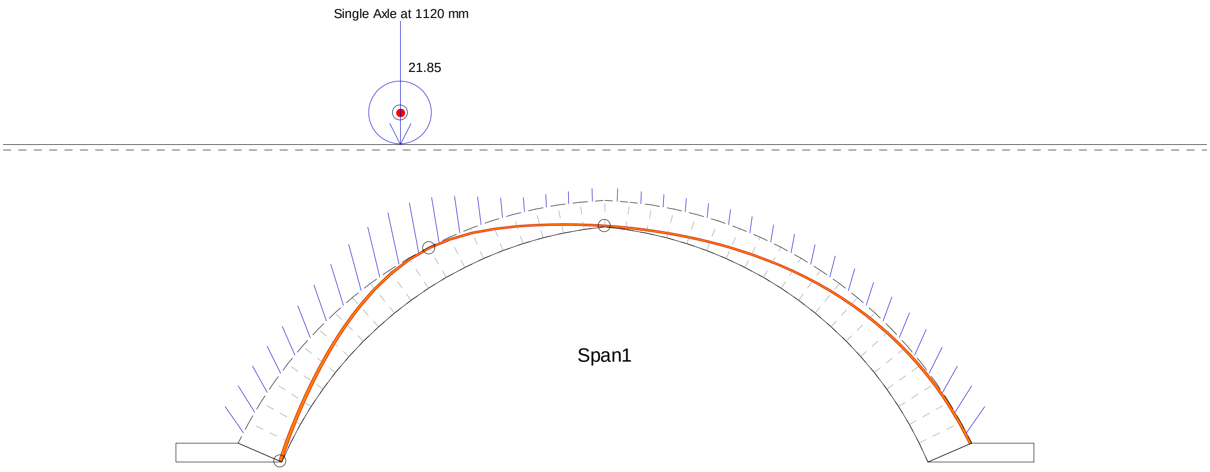
Here the ring is 250mm thick at the crown and 570 at the springing.

This load point is no longer the worst case, but a ring expanding to 580mm at the springing is OK even with the very conservative view of behaviour.

The Exe bridge is about 12 feet wide, which means it is not able to accommodate two vehicles side by side. We can therefore sensibly utilise the full width of the bridge for this one vehicle.



The outcome is to dramatically reduce the ring thicknening required.



The ring thickening to only 450mm is now sufficient.

Note that this ring thickening has a double effect because the thrust is allowed to rise under the load as well as at the springing.

In truth, this bridge is almost certainly filled with something very similar to concrete.

Company News

Hamish Harvey is now fully engaged with the business. Alongside Bill he is currently at work on a model for shallow tunnels which is nearly finished as a spreadsheet but which now needs to be converted to proper robust code.

Before that we were working on a program to help design Flexi-Arches and we will be returning to that early in the New Year. We plan to release that as a pay per use web app.